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DETERMINATION OF STRESSES AT A FIXED POINT WITH SYMMETRICAL IMPACT OF PLANE JETS TAKING ACCOUNT OF COMPRESSIBILITY, VISCOSITY, AND STRENGTH OF MATERIALS

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A method is suggested for approximate calculation of stresses at a fixed point for flow characterizing the existence of plane or axial symmetry, taking account of the effects of compressibility, viscosity, and strength. At a fixed point, additions to hydrodynamic pressure caused by the effects listed assuming smallness of them are calculated. The method for determining additions is based on use of an iteration method [1], where as a zero approximation, flow of an ideally incompressible fluid is used. The addition as a result of compressibility is calculated in an acoustic approximation, and models for an ideally elastoplastic medium and a Newtonian fluid are used in calculating the additions resulting from strength and viscosity, respectively. The procedure for determining corrections makes it possible to use more complex rheological models. A process is given in detail for calculating corrections for the problem of impact of plane jets. Results are given for the problem of steady-state penetration of a plane jet into a half-space. A correction is determined for the velocity of a fixed point.

The set of equations describing steady-state planar flow of a compressible medium characterized by a nonspherical stress tensor is written in the form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=\frac{\nu}{\rho_0}\frac{\partial p}{\partial x}+\left(\frac{1}{\rho_0}-\frac{\nu}{\rho_0}\right)\left(\frac{\partial s_{xx}}{\partial x}+\frac{\partial s_{xy}}{\partial y}\right),$$
$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+\frac{1}{\rho_0}\frac{\partial p}{\partial y}=\frac{\nu}{\rho_0}\frac{\partial p}{\partial y}+\left(\frac{1}{\rho_0}-\frac{\nu}{\rho_0}\right)\left(\frac{\partial s_{xy}}{\partial x}+\frac{\partial s_{yy}}{\partial y}\right), \quad (1)$$
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\frac{1}{\nu-1}\left(u\frac{\partial \nu}{\partial x}+v\frac{\partial \nu}{\partial y}\right),\quad \nu=1-\rho_0/\rho,$$

where x and y are coordinates; u and v are components of the velocity vector; ρ is density, ρ_0 is initial density; p is pressure; s_{ij} are stress deviator components. Set (1) is written so that the right-hand parts of the equations are small if the effects of compressibility, viscosity, and strength are small. Assuming smallness for these effects in accordance with the iteration method in [1] at first flow is determined for the zero approximation, i.e., set (1) is resolved with zero right-hand parts. Then the solution of the zero approximation, together with the rheological model for the medium connecting stress-tensor components with components of the strain tensor, is used in order to calculate the right-hand parts of the set of equations of the first approximation

$$\frac{\partial (u^0u^1)}{\partial x}+v^0\frac{\partial u^1}{\partial y}+v^1\frac{du^0}{dy}+\frac{1}{\rho_0}\frac{\partial (p^0+p^1)}{\partial x}=\frac{1}{\rho_0}\left(\nu^0\frac{\partial p^0}{\partial x}+\frac{\partial s^0_{xx}}{\partial x}+\frac{\partial s^0_{xy}}{\partial y}\right),$$

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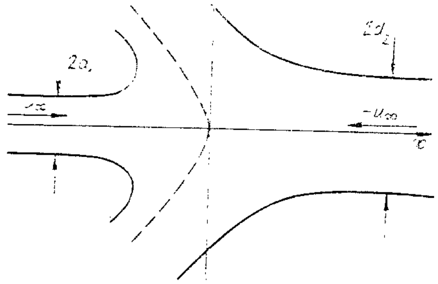

Fig. 1

$$u^0 \frac{\partial v^1}{\partial x} + u^1 \frac{\partial v^0}{\partial x} + \frac{\partial (v^0 v^1)}{\partial y} + \frac{1}{\rho_0} \frac{\partial (p^0 + p^1)}{\partial y} = \frac{1}{\rho_0} \left(\nu^0 \frac{\partial p^0}{\partial y} + \frac{\partial s^0_{xy}}{\partial x} + \frac{\partial s^0_{yy}}{\partial y} \right),$$
$$\frac{\partial u^1}{\partial x} + \frac{\partial v^1}{\partial y} = \frac{1}{\nu^0 - 1} \left(u^0 \frac{\partial \nu^0}{\partial x} + v^0 \frac{\partial \nu^0}{\partial y} \right), \quad \nu^0 = 1 - \rho_0/\rho(p^0) \tag{2}$$

(variables with index 1 pertain to the determination, and with an index 0 they are found from the solution of the zero approximation equation).

We consider the problem of head-on impact of plane jets with thickness $2d_1$ and $2d_2$ moving in opposite directions (see Fig. 1). The jets have the same density ρ_0 and velocity modulus u_∞. The steady-state solution for this problem is known [2]. We use it as a zero approximation in calculating stresses for a fixed point. Let $d_1 = d_2 = d$. Following [3] we integrate the first equation of set (2) along the surface of symmetry $y = 0$, taking account of boundary conditions for the first approximation $v^1(x, 0) = 0$, $u^1(0, 0) = 0$, $u^1(\infty, 0) = -u_\infty$, $p^1(\infty, 0) = 0$, the solution of the zero approximation $p^0(0, 0) = \rho_0 u_\infty^2/2$, $p^0(\infty, 0) = 0$, $v^0(x, 0) = 0$, and boundary conditions for the stress deviator component $s_{ij}(\infty, 0) = 0$. As a result of integration we obtain an expression for stress tensor components $\sigma_{xx}{}^1$ at a fixed point in a first approximation

$$-\sigma^1_{xx}(0, 0) = p^1(0, 0) - s^0_{xx}(0, 0) = p^0(0, 0) + \rho_0 \left(u_\infty^2/2 - P(p^0) \right) - \tag{3}$$
$$- \int_0^\infty \frac{\partial s^0_{xy}}{\partial y}\, dx, \quad P(p^0) = \int_0^{p^0} \frac{dp}{\rho(p)}.$$

The second term in the right-hand part of relationship (3) is a correction due to compressibility, and the third is due to strength and viscosity of the medium of the impacting jets. The method for calculating the correction for compressibility in the general case is given in [4], and for our purposes, assuming smallness of ν, it is possible to use an acoustic approximation $p = c_0{}^2(\rho - \rho_0)$ (c_0 is sound velocity). By calculating the corresponding integral we find the correction due to compressibility:

$$h_j = \rho_0 \left(\frac{u_\infty^2}{2} - c_0^2 \ln \left(1 + \frac{p^0(0, 0)}{\rho_0 c_0^2} \right) \right).$$

Considering that $p^0(0, 0) = \rho_0 u_\infty{}^2/2$, and by carrying out expansion into a series for small parameter $u_\infty{}^2/c_0{}^2$, we have

$$h_j = p^0(0, 0)\, u_\infty^2/(4c_0^2). \tag{4}$$

Now we determine the correction for a fluid with Newtonian viscosity $s_{xy}{}^0 = \mu(\partial u^0/\partial y + \partial v^0/\partial x)$ (μ is viscosity coefficient). First we note that, from the conditions of incompressibility, potentiality, and symmetry of flow for the zero approximation it is easy to find the subsequent relationships with $y = 0$:

$$v^0, \quad \frac{\partial v^0}{\partial x}, \quad \frac{\partial^2 v^0}{\partial x^2}, \quad \frac{\partial^2 v^0}{\partial y^2}, \quad \frac{\partial u^0}{\partial y}, \quad \frac{\partial^2 u^0}{\partial x \partial y} = 0,$$
$$\frac{\partial^2 v^0}{\partial x \partial y} = -\frac{\partial^2 u^0}{\partial x^2}, \quad \frac{\partial^2 u^0}{\partial y^2} = -\frac{\partial^2 u^0}{\partial x^2}, \quad \frac{\partial v^0}{\partial y} = -\frac{\partial u^0}{\partial x}. \tag{5}$$

For $d_1 = d_2 = d$, by using the general solution in [2] at the surface of symmetry y = 0 we obtain

$$x = -\frac{2d}{\pi}\left[\ln\left(\frac{u_\infty + u^0}{u_\infty - u^0}\right) + 2\,\mathrm{arctg}\left(\frac{u^0}{u_\infty}\right)\right]. \tag{6}$$

From (3), taking account of (5) and (6), we find the correction for stress as a result of viscosity

$$h_v = -\int_0^\infty \frac{\partial s_{xy}}{\partial y}\,dx = 2\mu\int_0^\infty \frac{d^2 u^0}{dx^2}\,dx = \mu\frac{\pi u_\infty}{4d}. \tag{7}$$

Now we move to determining the strength correction by using the solution of the zero approximation (6) and a model for an ideally elastoplastic medium:

$$\begin{gathered} \dot{s}_{ij} = 2g\varepsilon_{ij} \text{ in the region of elasticity.} \\ \varepsilon_{ij} = \lambda s_{ij}, \quad s_{ij}s_{ij} = 2Y_0^2/3 \text{ in the plastic region,} \\ \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \end{gathered} \tag{8}$$

Here $\dot{s}_{ij} = (d/dt)(s_{ij})$ is total derivative with respect to time; ε_{ij} are strain-rate tensor components; Y_0 is yield point; λ is coefficient of proportionality. It is noted that $\varepsilon_{ii}{}^0 = 0$ due to flow incompressibility for the first approximation and, therefore, the strain-rate deviator coincides with the strain-rate tensor. Calculation of strength corrections is carried separately in regions of elasticity and plasticity by omitting in subsequent calculations the upper index for solving the zero approximation. We designate in terms of x_* the coordinate of the boundary separating the elastic and plastic regions, and $-u_*$ is the velocity determined for this coordinate from (6). We find the coefficient of proportionality λ in the plastic region by using a rule for plastic flow and the fluidity criterion from (8):

$$\lambda = \frac{\sqrt{3}}{Y_0}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{1}{4}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]^{1/2}. \tag{9}$$

By substituting (9) in (8) and differentiating with respect to y, taking account of (5), we obtain

$$\frac{\partial s_{xy}}{\partial y} = -\frac{Y_0\dfrac{\partial^2 u}{\partial x^2}}{\sqrt{3}\left|\dfrac{\partial u}{\partial x}\right|}.$$

After integration we determine the correction in the region of plasticity

$$\int_0^{x_*} \frac{\partial s_{xy}}{\partial y}\,dx = \frac{Y_0}{\sqrt{3}}\ln\left(1 - \frac{u_*^4}{u_\infty^4}\right). \tag{10}$$

In the zone of elasticity Hooke's law $\dot{s}_{xy} = g(\partial u/\partial y + \partial v/\partial x)$ (g is shear modulus) is valid. After differentiating this relationship with respect to y, taking account of (5) and the steady state of flow, we obtain an equation for determining $\partial s_{xy}/\partial y$:

$$u\frac{\partial}{\partial x}\left(\frac{\partial s_{xy}}{\partial y}\right) - \frac{\partial u}{\partial x}\frac{\partial s_{xy}}{\partial y} + 2g\frac{\partial^2 u}{\partial x^2} = 0.$$

Its solution satisfying boundary condition $\partial s_{xy}/\partial y(\infty, 0) = 0$ has the form $\partial s_{xy}/\partial y = g[\pi \cdot (u_\infty^2 - u^2)u]/(2du_\infty^3)$. By integrating the solution found we determine the correction in the region of elasticity

$$\int_{x_*}^{\infty} \frac{\partial s_{xy}}{\partial y}\,dx = 2g\left[\ln\left(1+\frac{u_*^2}{u_\infty^2}\right)-\ln 2\right]. \qquad (11)$$

In order to find the velocity at the boundary of the region of elasticity we integrate expressions for s_{xx} and s_{yy} along the trajectory in the elastic region taking account of boundary conditions $s_{xy}(x, 0)$, $s_{xx}(\infty, 0)$, and $s_{yy}(\infty, 0) = 0$. We obtain the connection of stresses with velocity at the boundary of the region $s_{xx} = -s_{yy} = -2g\ln(u_\infty/u_*)$. From the condition for achieving with stresses a ring of fluidity we find the velocity at the boundary of the elastic region

$$u_* = u_\infty \exp(-Y_0/(2\sqrt{3}g)). \qquad (12)$$

Relationships (10)-(12) give a solution of the problem for determining the strength correction for pressure at a fixed point. By laying out the solution in a series for small parameter Y_0/g, with an accuracy to terms of a higher order of smallness we find that

$$h_h = \frac{Y_0}{\sqrt{3}}\left[\ln\left(\frac{\sqrt{3}g}{2Y_0}\right)+1\right]. \qquad (13)$$

Thus, for the problem of impact of plane jets in opposite directions corrections have been determined for stress at a fixed point (4), (7), and (13) due to the existence of compressibility, viscosity, and strength, respectively.

The problem of steady-state penetration of a jet into a half-space is the limiting case of the problem of head-on impact of jets with $d_1 = d = \text{const}$ and $d_2 \to \infty$ (see Fig. 1). The solution of the problem at the surface of symmetry y = 0:

$$x = -\frac{2d}{\pi}\left[\ln\left(\frac{u_\infty+u}{u_\infty-u}\right)-\frac{2u_\infty}{u_\infty+u}-\frac{2u_\infty^2}{(u_\infty+u)^2}+4\right].$$

By integrating the first equation of set (2) along line y = 0 we obtain relationships for determining stresses at a fixed point:

$$-\sigma_{xx}^{j}(0, 0) = \frac{\rho_0 u_\infty^2}{2} + \rho_0\left(\frac{u_\infty^2}{2} - P(p)\right) + \int_{-\infty}^{0} \frac{\partial s_{xy}}{\partial y}\,dx,$$

$$-\sigma_{xx}^{h}(0, 0) = \frac{\rho_0 u_\infty^2}{2} + \rho_0\left(\frac{u_\infty^2}{2} - P(p)\right) - \int_{0}^{\infty} \frac{\partial s_{xy}}{\partial y}\,dx$$

(values with upper index j relate to the jet, and with h to the half-space). By following the procedure given above, we determine corrections for stresses at a fixed point for the flow in question:

$$h_j^j = h_j^h = p^0(0,0)\frac{u_\infty^2}{4c_0^2}, \quad h_v^j = h_v^h = \mu\frac{\pi u_\infty}{8d},$$

$$h_h^j = \frac{Y_0}{\sqrt{3}}\left[\ln\left(\frac{\sqrt{3}g}{4Y_0}\right)+1\right], \quad h_h^h = \frac{Y_0}{\sqrt{3}}\left[3\ln\left(\frac{2\sqrt{3}g}{Y_0}\right)-\ln 2+1\right]$$

(expressions for h_h^j and h_h^h were obtained by expanding into a series for small parameter Y_0/g). As can be seen, strength corrections of the first approximation for a jet and for a half-space are different. By requiring fulfillment of the condition $\sigma_{xx}^j(0, 0) = \sigma_{xx}^h(0, 0)$, this makes it possible to find the correction of the first approximation for velocity at a fixed point: $\Delta u = (h_h^h - h_h^j)/(2\rho_0 u_\infty)$.

We give, without derivation, the result for spatially axisymmetrical flow describing superposition of potentials for steady-state uniform flow and a source of arbitrary intensity. The strength correction at the stagnation point obtained by integrating with respect to steady-state flow along the axis of symmetry has the form $h_h^h = Y_0[\ln(3g/2Y_0) + 2/3]$.

We formulate assumptions making it possible to obtain the results given above: $u_\infty/c_0 \ll 1$ is smallness of the correction for compressibility, applicability of an acoustic approximation, $\mu/(\rho_0 u_\infty d) \ll 1$ is smallness of the viscosity contribution, $Y_0/(\rho_0 u_\infty^2) \ll 1$ is smallness of the correction due to strength. The assumption $Y_0/g \ll 1$ is immaterial, and it makes it possible to simplify the form of the final equations.

The approach developed for calculating corrections for the first approximation permits apparent generalization in the case of impact of jets with different strength properties since flow for the zero approximation does not depend on rheology of the jet materials.

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EXPERIMENTAL STUDY OF VISCOUS INSTABILITY IN A POROUS MEDIUM

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Viscous instability of the displacement front in a porous medium arises when a liquid with a higher viscosity is displaced by a less-viscous liquid or gas. A large number of theoretical and experimental studies have been made of the formation and development of the fingers of displacing liquid that are produced in the process (see the reviews in [1, 2]). The displacement stability condition for neutrally wetted porous media was first obtained in [3]. The flow of liquids in this case occurs in different regions (piston displacement) and the capillary forces are taken into account in the boundary conditions at the displacement front. The nonlinear stage of development of liquid instability in the case of piston displacement was studied experimentally and theoretically in a Hele—Shaw cell [4].

When a porous medium is wetted well by one of the liquids, the two liquids flow jointly in porous space throughout the entire displacement region [5]. The condition for the stability of the displacement front against small perturbations with allowance for two-phase flow was given in [6]. Elsewhere [7] we showed that, in the case of unstable displacement capillary forces, which cause return flows of liquid in regions of high saturation gradients of the displacing liquid, stabilize the length of the fingers. At the same time, experimental data on the structure and growth dynamics of fingers of the displacing liquid in a porous medium during developed two-phase flow are lacking at present.

In this communication we report the results of an experimental study of the distinctive features in the development of fingers of displacing liquid during unstable displacement under the conditions of developed two-phase flow in the displacement region. The experimental data obtained on the structure and growth dynamics of fingers are compared directly with the results of numerical calculations on the basis of the Masket—Leverett model.

The experiments were carried out on a rectangular model of a porous medium with a working part measuring 1 × 20 × 60 cm, arranged horizontally. The working part was filled with quartz sand, which was then vibrocompacted with the porous state completely saturated with water. This made it possible to obtain a homogeneous porous medium with permeability ~10 μm^2 and porosity $m = 0.4$. After the vibrocompaction, the porous medium was dried, vacuum

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